Mrs Israel
Mrs Kerr
Ms Lau
Ms Prosser
Mrs Semler

Name: ……………………………………..……….

Teacher:……………………………………………….

Pymble Ladies' College

HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

2016

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes.
- Working time – 2 hours.
- Write using pencil for Questions 1-10.
- Write using black or blue pen for Questions 11-14. Black pen is preferred.
- Board approved calculators may be used.
- A reference sheet is provided.
- In Questions 11-14, show relevant mathematical reasoning and/or calculations.

Total Marks – 70

Section I Pages 1-4

10 marks

- Attempt all Questions 1-10
- Allow about 15 minutes for this section

Section II Pages 5-12

60 marks

- Attempt Questions 11-14
- Allow about 1 hour 45 minutes for this section

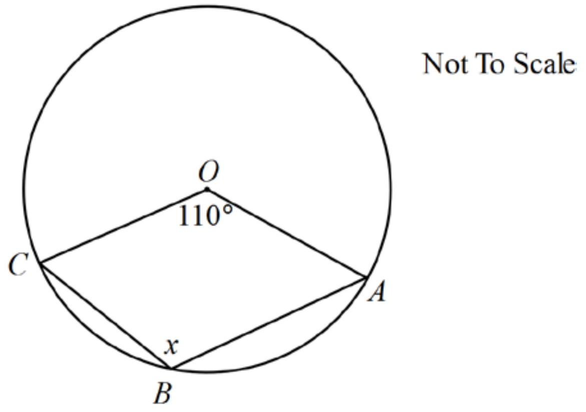
Mark	**/70**
Highest Mark	**/70**
Rank	

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section.

Use the multiple choice answer sheet for Questions 1-10.

1



What is the size of x?

(A) 70°

(B) 110°

(C) 125°

(D) 250°

2 Which geometric series has a limiting sum?

(A) $\sin\frac{\pi}{2} - \sin^2\frac{\pi}{2} + \sin^3\frac{\pi}{2} - \ldots\ldots$

(B) $\sin\frac{\pi}{6} + 4\sin^2\frac{\pi}{6} + 16\sin^3\frac{\pi}{6} + \ldots\ldots$

(C) $\tan\frac{\pi}{4} + \tan^2\frac{\pi}{4} + \tan^3\frac{\pi}{4} + \ldots\ldots$

(D) $\tan\frac{\pi}{6} + \frac{1}{2}\tan^2\frac{\pi}{6} + \frac{1}{4}\tan^3\frac{\pi}{6} + \ldots\ldots$

3 What are the domain and range for $y = \sin^{-1} x$?

(A) $-1 \le x \le 1;\ 0 \le y \le \pi$

(B) $-\frac{\pi}{2} \le x \le \frac{\pi}{2};\ -1 \le y \le 1$

(C) $-1 \le x \le 1;\ -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

(D) $0 \le x \le \pi;\ -1 \le y \le 1$

4 What are the coordinates of the focus of the parabola $12y = x^2 - 6x - 3$?

(A) $(-3,1)$

(B) $(3,-4)$

(C) $(3,-1)$

(D) $(3,\ 2)$

5 The roots of the polynomial $P(x) = 2x^3 - 4x + 1$ are α, β and γ.
What is the value of $\alpha\beta(\alpha + \beta)$?

(A) 1

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 2

6 Which expression is equal to $\sqrt{3}\cos x - \sin x$?

(A) $2\cos\left(x+\frac{\pi}{6}\right)$

(B) $2\cos\left(x+\frac{\pi}{3}\right)$

(C) $2\cos\left(x-\frac{\pi}{6}\right)$

(D) $2\cos\left(x-\frac{\pi}{3}\right)$

7 If $\frac{dP}{dt} = 0\cdot 4(P-20)$, and $P = 60$ when $t = 0$, which of the following is an expression for P ?

(A) $P = 40 + 20e^{0.4t}$

(B) $P = 60 + 20e^{0.4t}$

(C) $P = 20 + 40e^{0.4t}$

(D) $P = 20 + 60e^{0.4t}$

8 Differentiate $e^{2x}\cos 3x$ with respect to x.

(A) $e^{2x}\left(2\cos 3x - 3\sin 3x\right)$

(B) $-6e^{2x}\sin 3x$

(C) $e^{2x}\left(2\cos 3x - \sin 3x\right)$

(D) $e^{2x}\left(2\cos 3x + 3\sin 3x\right)$

9 Which is the solution $\frac{1}{x} \leq \frac{2}{1+2x}$?

(A) $\frac{1}{2} \leq x \leq 0$

(B) $x < -\frac{1}{2}, x > 0$

(C) $-\frac{1}{2} < x < 0$

(D) $-\frac{1}{2} > x > 0$

10 By considering the binomial expansion of $(1+x)^{100} - (1-x)^{100}$,

what is the value of $100_{C_3} + 100_{C_5} + ...+100_{C_{99}}$?

(A) 2^{100}

(B) 2^{99}

(C) $2^{100} - 100$

(D) $2^{99} - 100$

Section II

60 marks
Attempt Questions 11-14

Answer each question in a SEPARATE writing booklet. Extra booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a Separate Booklet. **Marks**

(a) Consider the function $f(x) = x^3 + 2x^2 - 5x - 6$.

(i) Show that $x - 2$ is a factor of $f(x)$. **1**

(ii) Hence, solve $x^3 + 2x^2 - 5x - 6 > 0$. **2**

(b) Use the substitution $u = \sqrt{x}$ to evaluate $\int_1^9 \frac{dx}{x + \sqrt{x}}$. **3**

(c) Find the constant term in the expansion of $\left(x^2 + \frac{2}{x}\right)^3$. **2**

(d) (i) Prove $\sin 2x - \tan x \cos 2x = \tan x$. **2**

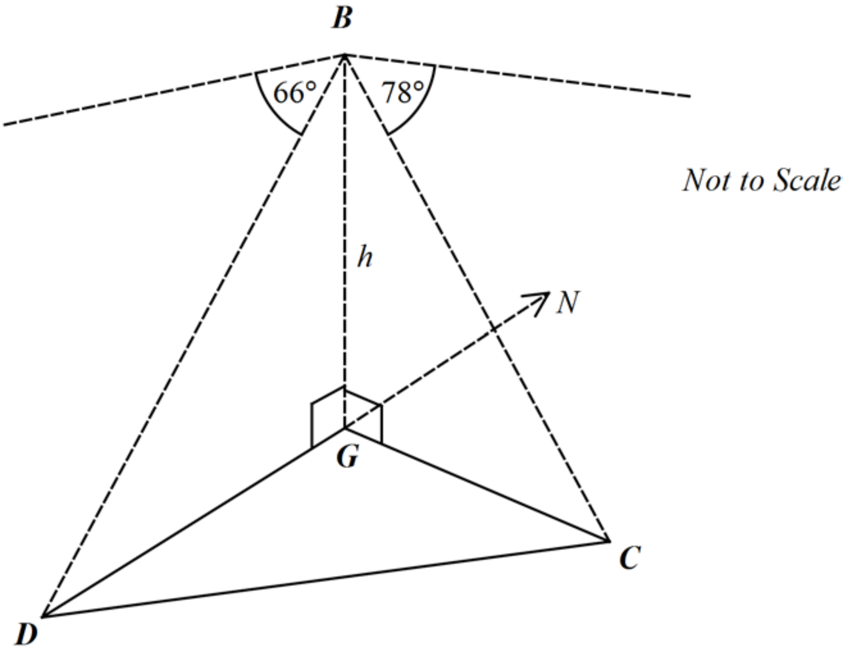
(ii) Hence, show that $\tan \frac{3\pi}{8} = \sqrt{2} + 1$. **2**

Question 11 continues on page 6

(e) *BG* is the height, *h*, of a balloon above the ground. From the balloon *B*, a dog is observed at *D* with an angle of depression of $66°$, and a cat with an angle of depression of $78°$. The dog and cat are 100 metres apart. **3**

The dog is due south of the balloon, and the bearing of the cat is $120°$ from the balloon.

What is *h*, the height of the balloon (correct to the nearest metre)?



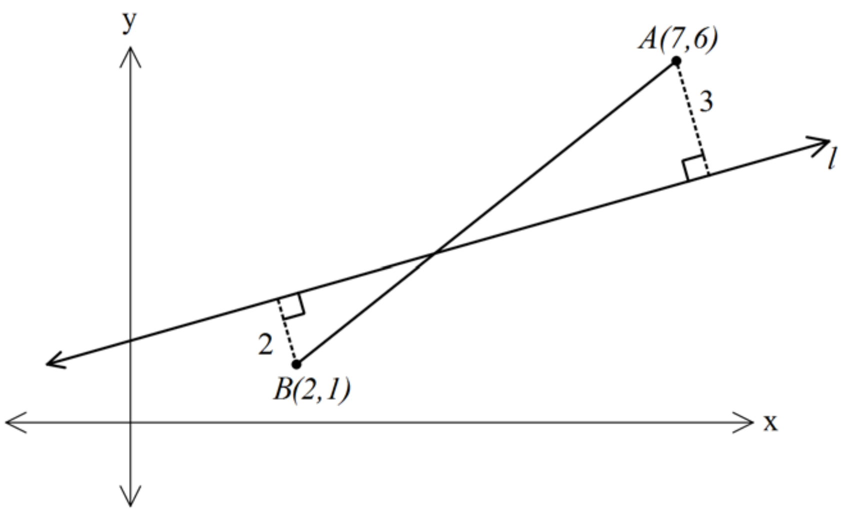
End of Question 11

Question 12 (15 marks) Use a Separate Booklet. **Marks**

(a) (i) Show that the equation $x^3 - 2x^2 - 2$ has a root α between 2 and 3. **1**

(ii) Use Newton's method with the initial approximation $x_0 = 3$ to find the value of α correct to 3 significant figures. **2**

(b)



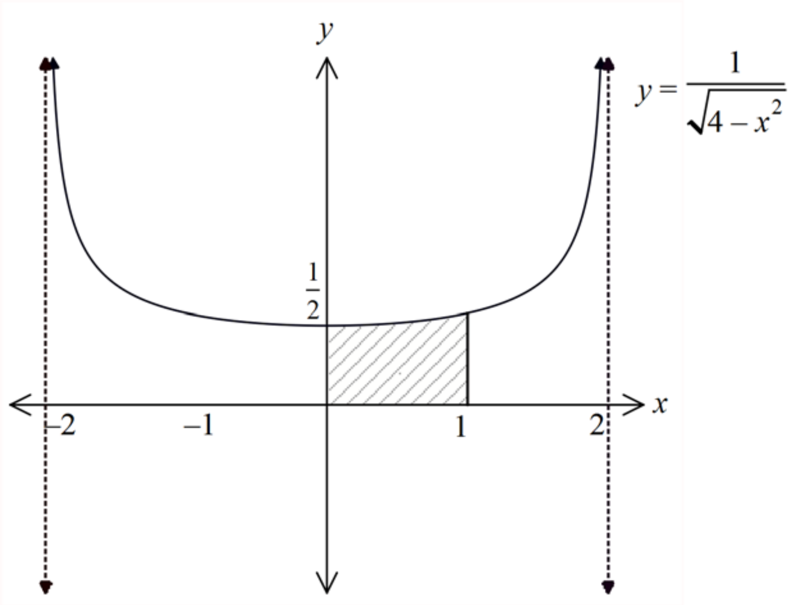
The points $A(7,6)$ and $B(2,1)$ are 3 units and 2 units respectively from line *l* and are on opposite sides of *l*. **2**

Find the coordinates of the point where the interval *AB* crosses line *l*.

Question 12 continues on page 8

(c) Show that $\dfrac{\sin 3A}{\sin A} - \dfrac{\cos 3A}{\cos A} = 2$ **3**

(d)



In the diagram above the curve $y = \dfrac{1}{\sqrt{4-x^2}}$ is sketched showing vertical asymptotes at $x = -2$ and $x = 2$. **3**

Find the exact area of the shaded region bounded by the curve, the line $x = 1$ and the coordinate axis.

(e) The acceleration of a particle moving along the x axis is given by $\ddot{x} = x - 2$, where x is its displacement from the origin O after t seconds. Initially the particle is at rest at $x = 3$.

(i) Show that its velocity at any position is given by $v^2 = (x-1)(x-3)$. **2**

(ii) Explain, using mathematical reasoning, why the particle can never move to the left of it's initial position. **2**

End of Question 12

Question 13 (15 marks) Use a Separate Booklet. **Marks**

(a) (i) Show that $\frac{d}{dx}\left(x\tan^{-1}x-\frac{1}{2}\ln\left(1+x^2\right)\right)=\tan^{-1}x$. **2**

(ii) Hence find the area between the curve $y=\tan^{-1}x$ and the x-axis for $0\le x\le 1$. **2**

(b) Use mathematical induction to prove that 2^n-1 is divisible by 3 for all **even** integers $n\ge 2$. **3**

(c) The rise and fall of the tide is assumed to be simple harmonic, with the time between low and high tide being six hours.
The water depth at a harbour entrance at high and low tides are 16 metres and 10 metres respectively.

(i) Show that the water depth, y metres, in the harbour is given by **2**

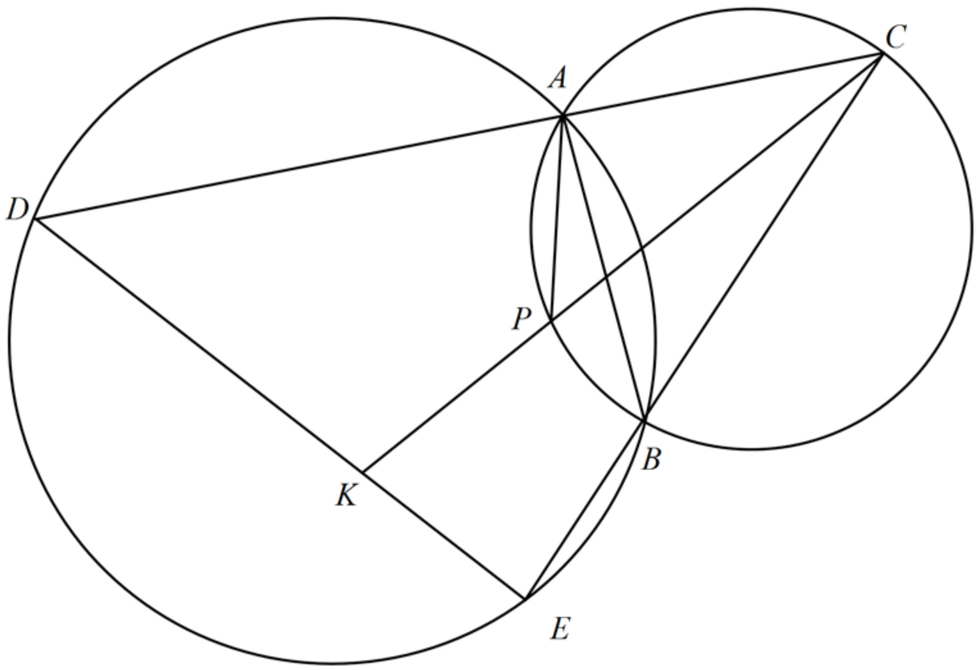
$$y=13-3\cos\left(\frac{\pi t}{6}\right)$$

where t is the number of hours after high tide.

(ii) On the morning a ship is to sail into the harbour entrance, low tide is at 8 am. If the ship requires a water depth of 12 metres in which to sail, what is the earliest time the ship can enter the harbour after 8am? **2**

Question 13 continues on page 10

(d)



The circles intersect at *A* and *B*. The lines *DAC*, *EBC*, *KPC* and *DKE* are all straight lines.

(i) Copy or trace the diagram into your answer booklet.

(ii) Give a reason why $\angle CBA = \angle CPA$. **1**

(iii) Hence or otherwise, show that *PADK* is a cyclic quadrilateral. **3**

End of Question 13

Question 14 (15 marks) Use a Separate Booklet. **Marks**

(a) A cup of hot chocolate at temperature $T°$ Celsius cools according to the differential equation

$$\frac{dT}{dt} = -k\left(T - R\right).$$ where t is time elapsed in minutes,

R the temperature of the room in degrees Celsius and k is a positive constant.

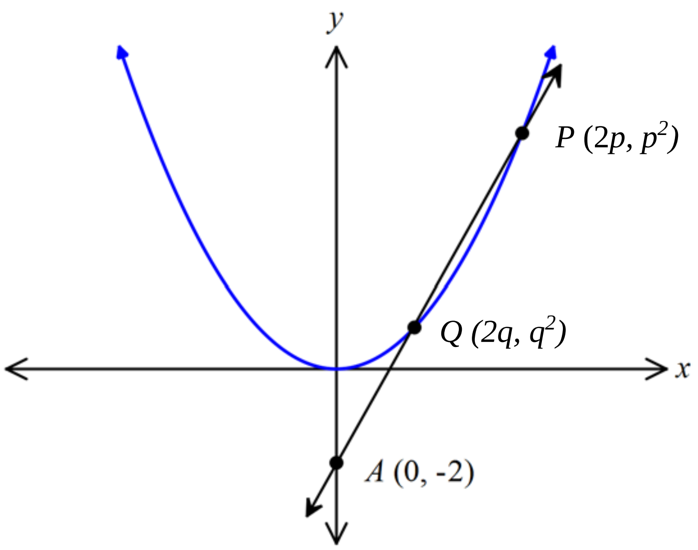
(i) Show that $T = R + Ae^{-kt}$, where A is a constant, is a solution of the differential equation. **1**

(ii) A cup of hot chocolate which is $70°C$ is placed in a room with temperature $20°C$. After 10 minutes, the chocolate has cooled to $35°C$. Find the exact value of k. **2**

(b) A rectangle is expanding in such a way that at all times it is twice as long as it is wide. If its area is increasing at a rate of 18 cm^2/s, find the rate at which its perimeter is increasing at the instant its width is 2 metres. **3**

Question 14 continues on page 12

(c)



(i) State the equations of the normals to the parabola $x = 2t,\ y = t^2$ at the points $P\left(2p, p^2\right)$ and $Q\left(2q, q^2\right)$, where $p \neq q$. **1**

(ii) Hence, show that these normals intersect at the point $R(X, Y)$ where $X = -pq(p+q)$ and $Y = (p+q)^2 - pq + 2$. **2**

(iii) If the chord *PQ* has gradient *m* and passes through the point $A(0, -2)$ find, in terms of *m*, the equation of *PQ* and hence show that *p* and *q* are the roots of the equation $t^2 - 2mt + 2 = 0$. **2**

(iv) By considering the sum and the product of the roots of this quadratic equation, show that the point *R* lies on the original parabola. **2**

(v) Find the least value for m^2 for which *p* and *q* are real. Hence find the set of possible values of the *y* coordinate of *R*. **2**

End of paper

Extension 1 2016 Trial solutions

MC

1. C	4. D	7. C	9. C
2. D	5. C	8. A	10. D
3. C	6. A		

Question 11:

(a)(i) $f(x) = x^3 + 2x^2 - 5x - 6$ $(x-2)$

$f(2) = (2)^3 + 2(2)^2 - 5(2) - 6$

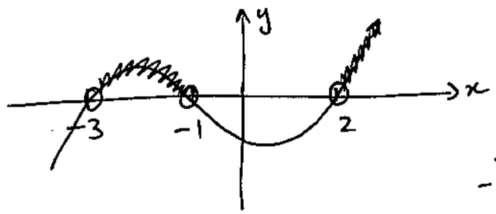
$= 0$ $\therefore x-2$ is a factor (1)

(ii)

$$\begin{array}{r} x^2+4x+3 \\ x-2 \overline{) x^3+2x^2-5x-6} \\ x^3-2x^2 \\ \hline 4x^2-5x \\ 4x^2-8x \\ \hline 3x-6 \\ 3x-6 \\ \hline 0 \end{array}$$

$f(x) = (x-2)(x^2+4x+3)$

$= (x-2)(x+3)(x+1)$ 1 mark



(2)

$\therefore -3 < x < -1, \; x > 2$ 1 mark

(b) when $x = 9$ $u = \sqrt{9} = 3$

$x = 1$ $u = \sqrt{1} = 1$

if $u = \sqrt{x}$ $\therefore u^2 = x$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$du = \frac{dx}{2\sqrt{x}}$

$dx = 2u.du$

1 mark

$$\int_1^9 \frac{dx}{x+\sqrt{x}} = \int_1^3 \frac{2u\,du}{u^2+u} \quad 1$$

$$= \int_1^3 \frac{2\,du}{u+1}$$

$$= 2\left[\ln(u+1)\right]_1^3$$

$$= 2\ln 4 - 2\ln 2$$

$$= \ln 4 \quad 1$$

(3)

Q11 (c) $^{n}C_r\, x^{n-r}\, y^{r}$ $\left(x^2 + \frac{2}{x}\right)^3$

$^{3}C_r \left(x^2\right)^{3-r} \left(\frac{2}{x}\right)^r$

$^{3}C_r\, x^{6-2r}\, 2^r\, x^{-r}$

$^{3}C_r\, 2^r\, x^{6-3r}$ 1

for constant $6-3r=0$

$r=2$

$\therefore\ ^{3}C_2\, 2^2 = 12$ 1 $\therefore$ constant is 12 (2)

d) (i) $\sin 2x - \tan x \cos 2x = \tan x$

LHS $= 2\sin x \cos x - \tan x(1 - 2\sin^2 x)$ 1

$= \frac{2\sin x \cos^2 x}{\cos x} - \frac{\sin x}{\cos x} + \frac{2\sin x \sin^2 x}{\cos}$

$= \frac{2\sin x(1-\sin^2 x) - \sin x + 2\sin^3 x}{\cos x}$

$= \frac{2\sin x - 2\sin^3 x - \sin x + 2\sin^3 x}{\cos x}$ 1

$= \frac{\sin x}{\cos x}$

$= \tan x$ (2)

$=$ RHS

Q11 d (ii) $\tan x = \sin 2x - \tan x \cos 2x$

$\tan\left(\frac{3\pi}{8}\right) = \sin 2\left(\frac{3\pi}{8}\right) - \tan\left(\frac{3\pi}{8}\right)\cos 2\left(\frac{3\pi}{8}\right)$

$= \sin\frac{3\pi}{4} - \tan\frac{3\pi}{8}\cos\frac{3\pi}{4}$

$= \frac{1}{\sqrt{2}} - \tan\frac{3\pi}{8}\cdot\left(-\frac{1}{\sqrt{2}}\right)$ } 1

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\tan\frac{3\pi}{8}$

$\tan\frac{3\pi}{8} - \frac{1}{\sqrt{2}}\tan\frac{3\pi}{8} = \frac{1}{\sqrt{2}}$

$\tan\frac{3\pi}{8}\left(1 - \frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}$

$\tan\frac{3\pi}{8} = \frac{1}{\sqrt{2}} \div \left(1 - \frac{1}{\sqrt{2}}\right)$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}-1}$

$= \frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$ } 1

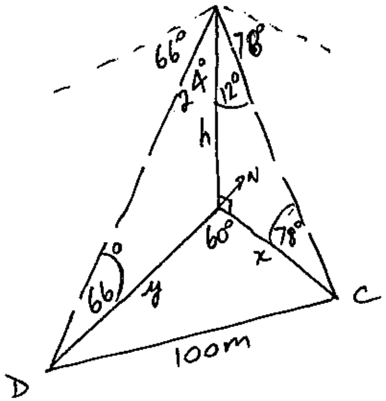
$= \frac{\sqrt{2}+1}{2-1}$

$1 - \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{2}}{\sqrt{2}} - \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{2}-1}{\sqrt{2}}$

$\therefore \tan\frac{3\pi}{8} = \sqrt{2}+1$ (2)



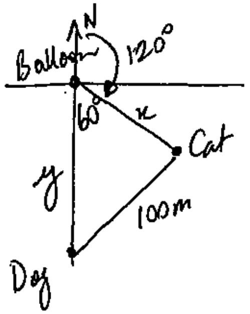
$\tan 12° = \frac{x}{h}$

$x = h\tan 12°$

$\tan 24° = \frac{y}{h}$

$y = h\tan 24°$ } 1

We want x, y on top



$100^2 = x^2 + y^2 - 2xy\cos 60°$

$100^2 = h^2\tan^2 12° + h^2\tan^2 24° - 2x\, h\tan 12°\, h\tan 24° \times \frac{1}{2}$ — 1

$100^2 = h^2(\tan^2 12° + \tan^2 24° - \tan 12° \tan 24°$

$h^2 = \frac{100^2}{\tan^2 12° + \tan^2 24° - \tan 12° \tan 24°}$

$h = 259.261....$

∴ height is 259 metres — 1 ③

QUESTION 12:

a) (i) let $f(x) = x^3 - 2x^2 - 2$

$f(2) = (2)^3 - 2(2)^2 - 2$
$= -2$
< 0

$f(3) = (3)^3 - 2(3)^2 - 2$
$= 7$
> 0

(1)

since $f(x)$ changes sign between $x=2$ and $x=3$ and $f(x)$ is continuous there is a root in the interval.

(ii) $f'(x) = 3x^2 - 4x$

$f'(3) = 3(3)^2 - 4(3)$
$= 15$

$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

$= 3 - \dfrac{7}{15}$

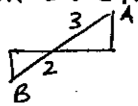
$= 2.5\dot{3}$

$= 2.53$ (3 sig fig)

(2)

b) Triangles are similar as equiangular $\therefore$ sides in same ratio

ratio $= \overset{m}{3} : \overset{n}{2}$ $A(7,6)$ $B(2,1)$



$x = \dfrac{2\times 7 + 3\times 2}{2+3}$
$= 4$

$y = \dfrac{2\times 6 + 3\times 1}{2+3}$
$= 3$

$\therefore$ pt is $(4, 3)$

This was marked generously but students should note that they should state the function is continuous in the interval.

This was well done by most.

Many students could not identify the link to dividing an interval internally in a ratio. Those who did were usually successful.

(c) $\frac{\sin 3A}{\sin A} - \frac{\cos 3A}{\cos A} = 2$

$$LHS = \frac{\sin(2A+A)\cos A}{\sin A \cos A} - \frac{\cos(2A+A)\sin A}{\sin A \cos A}$$

$$= \frac{\cos A(\sin 2A\cos A + \cos 2A \sin A) - \sin A(\cos 2A\cos A - \sin 2A \sin A)}{\sin A \cos A}$$

$$= \frac{\sin 2A\cos^2 A + \cos 2A \sin A\cos A - \cos 2A \sin A\cos A + \sin 2A \sin^2 A}{\frac{1}{2}\sin 2A}$$

$$= \frac{2(\sin 2A\cos^2 A + \sin 2A \sin^2 A)}{\sin 2A}$$

$$= \frac{2\sin 2A(\cos^2 A + \sin^2 A)}{\sin 2A} \quad \text{as } \sin^2 A + \cos^2 A = 1.$$

$= 2 \times 1$

$= 2$

(3)

Students are reminded to not take short cuts in a "prove" question.

d) $y = \frac{1}{\sqrt{4-x^2}}$

$$\int_0^1 \frac{1}{\sqrt{4-x^2}}\,dx$$

$$= \left[\sin^{-1}\left(\frac{x}{2}\right)\right]_0^1$$

$$= \sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(0)$$

$$= \frac{\pi}{6} - 0$$

$$= \frac{\pi}{6} \text{ units}^2$$

(3)

Generally well done.

e) (i) $\ddot{x} = x - 2$ when $t=0$ $x=3$ $v=0$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = x-2$$

$$\frac{1}{2}v^2 = \int (x-2)\,dx$$

$$\frac{1}{2}v^2 = \frac{x^2}{2} - 2x + c \qquad 1$$

when $x=3$ $v=0$

$$0 = \frac{3^2}{2} - 2\times 3 + c$$

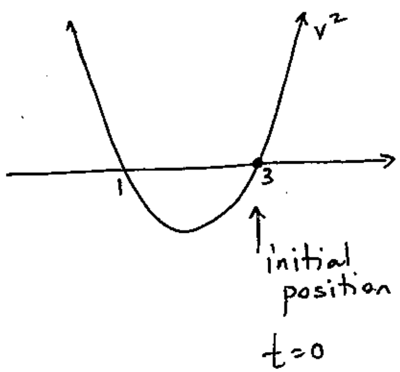
$$\therefore c = 3/2$$

$$\frac{1}{2}v^2 = \frac{x^2}{2} - 2x + \frac{3}{2}$$

$$v^2 = x^2 - 4x + 3$$

$$v^2 = (x-3)(x-1) \qquad 1 \qquad (2)$$

(ii)



explanation of why particle can't move left of 3.....

1 – reasonable progress

must say $v > 0$ positive
initial position 3
acceleration positive

(2)

Generally well done

Mary struggled to coherently and clearly explain why the particle couldn't move to the left of its initial position.

(a)(i) $\frac{d}{dx}\left(x\tan^{-1}(x) - \frac{1}{2}\ln(1+x^2)\right)$

$u = x \quad v = \tan^{-1}(x)$

$u' = 1 \quad v' = \frac{1}{1+x^2}$

$\frac{d}{dx}() = x \cdot \frac{1}{1+x^2} + \tan^{-1}(x) \times 1 - \frac{1}{2} \cdot \frac{2x}{1+x^2}$ ✓

$= \frac{x}{1+x^2} + \tan^{-1}(x) - \frac{x}{1+x^2}$ ✓

$= \tan^{-1}(x)$ (2)

(ii) $A = \int_0^1 \tan^{-1}(x)\, dx$

$= \left[x\tan^{-1}(x) - \frac{1}{2}\ln(1+x^2)\right]_0^1$

$= \left[1.\tan^{-1}(1) - \frac{1}{2}\ln(1+1^2)\right] - [0]$ ✓

$= \tan^{-1}(1) - \frac{1}{2}\ln 2$

$= \frac{\pi}{4} - \frac{\ln 2}{2}$

$= \left(\frac{\pi - 2\ln 2}{4}\right)$ units^2 ✓ (2)

(b) statement $2^n - 1 = 3P$ P is an integer ; n is even $n \geq 2$

1: when $n=2$ $2^2 - 1 = 3$ which is divisible by 3

∴ true for $n=2$

2: assume true $n=k$

$2^k - 1 = 3P$ P is an integer k even ✓

∴ $2^k = 3P+1$

3: RTP $n = k+2$

$2^{k+2} - 1 = 3Q$ Q is an integer

$LHS = 2^{k+2} - 1$

$= 2^2 . 2^k - 1$

$= 2^2 . (3P+1) - 1$ from assumption ✓

$= 12P + 4 - 1$

$= 12P + 3$

$= 3(4P+1)$ which is divisible by 3 ✓ since P is an integer

4: Hence if the statement is true for n=k then it is also true for n=k+1. Since it is true for n=2 then it is also true for $n = 2+2 = 4$, $n = 4+2 = 6$ and so on for all positive <u>even</u> integral values of n by mathematical induction.

(a) (i) no marks

(ii) they are angles at the circumference standing on the same arc AC in same segment. (1)

(iii) let $\angle CBA = \angle CPA = x$

$\angle APK = 180 - x$ } angles on a straight line
$\angle ABE = 180 - x$ }

$\therefore \angle APK = \angle ABE$

ABED is a cyclic quadrilateral

So $\angle ADE + \angle ABE = 180°$ (opposite angles of cyclic quadrilateral ABED)

$\therefore \angle ADE + 180° - x = 180°$

$\angle ADE = x°$

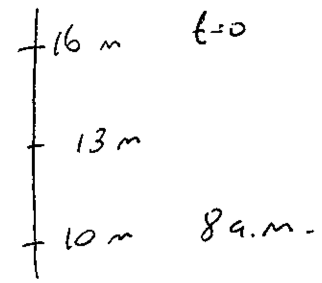
but similarily $\angle ADE$ and $\angle APK$ are opposite angles in a quadrilateral and

$\angle ADE + \angle APK = x° + 180° - x°$
$= 180°$

$\therefore$ as they are opposite angles in a quadrilateral and are supplementary PADK is a cyclic quadrilateral. (3)

(c)

(i)



$\frac{1}{2}$ period $=$ 6 hours

period $=$ 12 hours

$12 = \frac{2\pi}{n} \Rightarrow n = \frac{\pi}{6}$

Centre of motion $= \frac{10+16}{2}$
$= 13$ ✓

Amplitude $= 3$ ✓

✓

Using $y = y_0 - a\cos(nt)$

$= 13 - 3\cos\frac{\pi t}{6}$ AS REQ'D. (3)

(ii) When $y = 12$

$12 = 13 - 3\cos\frac{\pi t}{6}$

$3\cos\frac{\pi t}{6} = 1$ ✓

$\cos\frac{\pi t}{6} = \frac{1}{3}$ (2)

$\frac{\pi t}{6} = 1.2309...$ OR $5.05...$

$t = 2.35...$
$= 2$ hour $21.057...$ min ✓

THE EARLIEST TIME IS 10:21 a.m

QUESTION 14:

(a) (i) $\frac{dT}{dt} = -k\ Ae^{-kt}$ $\qquad Ae^{-kt} = T - R$

$= -k(T-R)$ ①

(ii) $t = 0 \quad T = 70^\circ \quad R = 20^\circ$

$t = 10 \quad T = 35^\circ$

So $T = 20 + 50e^{-kt}$

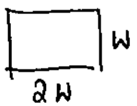
$35 = 20 + 50e^{-10k}$

$e^{-10k} = \frac{35-20}{50}$

$= 0.3$

$-10k = \ln 0.3$

$k = -\frac{\ln 0.3}{10}$ ②

b) 

$A = 2w^2$ $\qquad \frac{dA}{dt} = 18\text{cm}^2/\text{s}$

$P = 6w$

$\frac{dP}{dt} = \frac{dP}{dA} \times \frac{dA}{dt}$

$w = \frac{P}{6}$

$A = 2\left(\frac{P}{6}\right)^2$

$= \frac{P^2}{18}$ $\qquad$ when $w = \overset{\approx 2m}{\underset{cm}{200}}$ $P = 12000$ cm

$\frac{dA}{dp} = \frac{P}{9}$

$\therefore \frac{dP}{dt} = \frac{9}{P} \times 18$ ③

$= \frac{9}{12000} \times 18 = 0.135$ cm/s

(c) (i) $x + py = ap^3 + 2ap$

$a = 1 \quad x + py = p^3 + 2p \quad \text{---} ①$

similarily $x + qy = q^3 + 2q \quad \text{---} ②$ [1]

(ii) ① − ② from above

$$py - qy = p^3 - q^3 + 2p - 2q$$

$$(p-q)y = (p-q)(p^2+pq+q^2) + 2(p-q)$$

$$\therefore y = p^2 + pq + q^2 + 2$$

$$= (p+q)^2 - pq + 2 \quad \text{---} ③$$

sub ③ into ①

$$x + p\left[(p+q)^2 - pq + 2\right] = p^3 + 2p$$

$$x + p(p+q)^2 - p^2q + 2p = p^3 + 2p$$

$$x + p(p^2 + 2pq + q^2) - p^2q = p^3$$

$$x + p^3 + 2p^2q + pq^2 - p^2q = p^3$$

$$x + p^2q + pq^2 = 0$$

$$x = -p^2q - pq^2$$

$$= -pq(p+q)$$ [2]

iii) PQ $\overset{x\ \ y}{(0,-2)}$ gradient m $P(2p, p^2)$ $Q(2q, q^2)$

$\therefore$ PQ eq^n $y = mx - 2$

if P and Q lie on line then

$$p^2 = 2mp - 2$$

$$p^2 - 2mp + 2 = 0$$

similarily: $q^2 - 2mq + 2 = 0$

therefore p and q must be roots of

$t^2 - 2mt + 2 = 0.$ [2]

(vi) $t^2 - 2mt + 2 = 0$

as p and q are roots then

$p+q = 2m$

$pq = 2$

R: $x = -pq(p+q)$
$= -2(2m)$
$= -4m$

$y = (p+q)^2 - pq + 2$
$= (2m)^2 - (2) + 2$
$= 4m^2$

$\therefore R(-4m, 4m^2)$

parabola: $x = 2t \quad y = t^2$

$-4m = 2t$

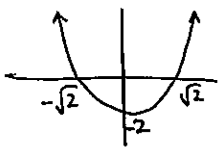
$\therefore t = -2m$

sub into y: $y = (-2m)^2$
$= 4m^2$ $\therefore$ R lies on parabola

[2]

(v) least value $\Delta = b^2 - 4ac$
$= (-2m)^2 - 4 \times 1 \times 2$
$= 4m^2 - 8$

real if $\Delta \geq 0$ $4(m^2-2) \geq 0$



$\therefore$ least value $m^2 - 2 = 0$
$m^2 = 2$

So given $y = 4m^2$

$y \geq 4 \times 2$

$y \geq 8$

[2]